The Semantics of ‘Zero’
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No standard semantic analysis for numerals is directly applicable to ‘zero’, and certain aspects of its behaviour set it aside from the other numerals. Treating ‘zero’ as a negative quantifier like ‘no’ is also problematic. I suggest an analysis of ‘zero’ that will give it a regular numeral semantics but still account for the peculiarities of its behaviour.

‘Zero’ vs. numerals. A common view of numeral semantics treats them as predicates of type \langle et \rangle (‘#’ stands for a function taking a set and returning its cardinality) (Rothstein 2016 is a recent implementation): \[ \text{three} = \lambda x . \#x = 3. \] ‘Three’ can combine intersectively (by Predicate Modification) with a noun (also \langle et \rangle), the result being those members of the nominal denotation that have three atomic elements: \[ \text{three people} = \lambda x . \#x = 3 & \text{people}(x). \]

In order for ‘three students’ to be used in argument position, existential quantification is introduced – usually, with a silent existential determiner A:

1. \[[A \text{three people arrived}] = \exists x . \#x = 3 & \text{people}(x) & \text{arrived}(x)\]

This treatment wouldn’t work for ‘zero’. The intersection will be either empty of the empty set (similarly for \langle et, et \rangle semantics of numerals, as in Ionin and Matushansky 2006). The silent existential determiner applied to an empty set results in a counter-intuitive meaning:

2. \[[A \text{zero people arrived}] = \exists x . \#x = 0 & \text{people}(x) & \text{arrived}(x)\]

(2) predicates \text{people} and \text{arrived} over the empty set – but any other predicate is true of the empty set as well, trivially. However, intuitively, (2) can be true or false depending on the situation. Changing the ‘= n’ to ‘\geq n’ is not a solution – in upper-bound readings, the > n sets will have to be eliminated anywhere. Moreover, upper-boundedness of ‘zero’ is even more persistent than with other numerals, and can’t be cancelled:

3. Three / # Zero students attended the conference, possibly more.

Finally, a type \langle n \rangle analysis runs into the same problem. Under this view, a numeral combines with a noun via mediation of a silent item \text{many} (Hackl 2000):

4. \[\text{many} = \lambda n . \lambda P, \lambda Q. \exists x . \#x = n & P(x) & Q(x)\]

Here, again, predication over the empty set is inevitable, and the problem reemerges.

‘Zero’ vs. ‘no’. One solution is to say that ‘zero’ is not a numeral at all. It could be treated as a negative quantifier akin to ‘no’: \[ [\text{zero}] = [\text{no}] = \lambda P, \lambda Q . P \cap Q = \emptyset. \]

This would solve the problem of predicking over the empty set, and would explain its upper-boundedness (3). However, ‘zero’ and ‘no’ are different in several ways. First, ‘no’, unlike ‘zero’, licenses strong NPIs (Gajewski 2007; Collins and Postal 2014).

5. a. No student / * Zero students have visited me in years.
   b. No student / * Zero students like semantics, either.

This is unexpected, as ‘zero’ and ‘no’ both create anti-additive environments (Gajewski 2007):

6. No / Zero boy(s) danced or smoked. \implies No / Zero boy(s) danced and no / zero boy(s) smoked.

Additionally, ‘zero’ (unlike ‘no’) doesn’t license exception phrases or bound readings of pronouns:

7. a. She drank no/*zero martinis, not even weak ones. (cf. Postal 2004)
   b. No/*zero students but Bill / except for Bill came. (cf. Moltmann 1995)

8. [No girl], / * [Zero girls], brought her friend to school.

This makes a quantifier analysis for ‘zero’ harder to formulate. Note that with respect to (7) and (8), ‘zero’ patterns with other numerals (NPIs excluded for other reasons):

   b. *[Three girls], brought her friend to school.

These observations made (Gajewski 2007) suggest that ‘zero’ is ‘just another number, like sixty four’ after all: the grammar is blind to the mathematical aspects of linguistic meaning, and for this reason, ‘zero’ it doesn’t license strong NPIs. One concern is that it’s an unwelcome restriction on the generality of the relation between anti-additivity and strong NPI licensing; second, problems inherent upper-boundedness of ‘zero’ remains unexplained.
Analysis. I suggest that ‘zero’ denotes a number, 0, and has semantic type \( \langle n \rangle \) – a type of denotation available for other numerals as well: \([\text{zero}] = 0\). This semantics allows ‘zero’ to be used in mathematical contexts in the same way as other numerals can be used: ‘Zero plus three minus two makes one’ (see Rothstein 2016).

However, unlike other numerals under some analyses, ‘zero’ doesn’t have an \( \langle et \rangle \) or \( \langle et, et \rangle \) counterpart. It also can’t combine with the standard silent \( \text{many} \) as in (4) because this would give rise to trivial truth-conditions. I suggest that in order for ‘zero’ to be used prenominally, a different version of \( \text{many} \) is needed. This new version would only differ from the standard one in the denotation of the noun it combines with – instead of \( \langle et \rangle \), it combines with the corresponding kind, type \( k \). This new \( \text{many}, \text{many}_k \), counts the realizations of the corresponding kind for which the predicate denoted by the rest of the sentence is true, and states that this number is equal to \( n \):

\[
\text{many}_k = \lambda n. \lambda x. \lambda Q. \eta((\langle x : Q(x) \rangle) = n)
\]

I remain agnostic as to whether the noun combining with \( \text{many} \) or is type-shifted from \( \langle et \rangle \) by a \( \cap \)-operator (Chierchia 1998 a.o.). The denotation in (10), when combined with \([\text{zero}]\), does not involve checking whether a predicate truthfully applies to an empty set, which was the source of the problem with other denotations. \( \text{many}_k \) plays a role similar to one sometimes assigned to classifiers (Chierchia 1998 a.o.) – taking the kind denotation of a noun and allowing it to combine with the numeral that counts realizations of this kind.

Notice that nothing precludes actually replacing the regular \( \text{many} \) with \( \text{many}_k \) for all numerals. However, this would leave the contrast in upper-boundedness (3) unexplained. According to (Landman 2004 a.o.), ‘at least’ readings of numerals are a result of the interaction between the ‘exactly’ meaning of the numeral and existential quantification, as in (1): that there is a group of people such that they arrived and that the cardinality of this group is three is compatible with there being other groups with higher cardinalities. (10) does not involve existential quantification over domain of individuals, and this source of ‘at least’ readings is blocked. I suggest that both \( \text{many} \) and \( \text{many}_k \) are available in English for almost all numerals (except for ‘zero’). In languages lacking prenominal use of ‘zero’ (Western Armenian, see Bale and Khanjian 2014), \( \text{many}_k \) does not exist.

This accounts for all differences between ‘zero’ and other numerals and for all differences between ‘zero’ and ‘no’, except for the NPI data. I suggest that ‘zero’ is part of the Horn scale of numerals, and constitutes its scalar endpoint. Following (Spector 2013), I assume that numerals are inherently focussed in that they automatically invoke their alternatives whenever used. I also propose that ‘zero’, being an endpoint scalar item, is always accompanied by a silent concessive focus particle akin to ‘even’. This scalar particle brings in presuppositions very similar to ‘even’ – it presupposes that other alternatives are more likely (or less noteworthy) (Crnič 2011).

Given this presupposition, sentences with ‘zero’ are still Strawson downward-entailing (when the presupposition is satisfied, inferences from sets to subsets are supported, see von Fintel 1999), which is enough for weak NPI licensing. However, strong NPIs are sensitive to anti-additivity rather than Strawson anti-additivity or (Strawson) downward-entailment (Gajewski 2007). This means that if a context is Strawson AA (AA ignoring presuppositions) but the presuppositions themselves are not AA, strong NPIs are blocked (as with ‘only’). I suggest that presuppositions in sentences with ‘zero’ are not AA, and this blocks strong NPIs. The presupposition in (11b) states that of each of the alternatives, there is a degree of likelihood (or noteworthiness) s.t. whatever this degree is, it’s higher (lower for noteworthiness) than that for ‘zero’. Whatever the alternative or the degree (unless it’s ‘zero’ and 0, respectively), the AA inference does not go through:

\[
(12) \text{It is } d\text{-likely that } n \text{ students smoke or dance.}
\]

Thus, although entailment patterns of sentences with ‘zero’ are similar to ones with ‘no’, the presuppositions of the former are responsible for the differences in their NPI-licensing profiles.