Rigidity and Distributivity in Plural Predicate Logic
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Indefinites and cardinal NPs are known to differ from bare NPs (bare plurals and mass nouns) in certain scopal environments. For example, only bare NPs can readily co-occur in the scope of for-adverbials. Unlike the NPs in (1a), those in (1b) pick out the same apples or amount of applesauce throughout the target hour; given the semantics of eat, this leads to deviance (Kritka 1998).

(1) a. John ate \{ apples / applesauce \} for an hour.
   b. *John ate \{ an apple / some apples / two apples / less than three apples \} for an hour.

As (2a)-(2b) show, the same patterns occur with spatial measure adverbials and indefinite quantity adverbials. Similarly, in (2c), a habitual interpretation is preferred only with the bare nominal that can pick out multiple articles. A further contrast crops up with all, as witnessed by (3a), where a cumulative reading is available, and (3b) where only a distributive reading is available.

(2) a. \{ Trees / #Some trees \} grow for miles around this castle. \hspace{1cm} (Moltmann 1991)
   b. He read (poetry / #something) a lot. \hspace{1cm} (Mittwoch 1982)
   c. John wrote \{ copy / an article \} for the Times. \hspace{1cm} (Mittwoch 1982)

(3) a. All the boys dated chemistry majors. \hspace{1cm} (Zweig 2009)
   b. All the boys dated \{ 5 / less than 5 / several \} chemistry majors. \hspace{1cm} (Zweig 2009)

Rigidity Thesis: We posit rigidity as a fundamental functional distinction between indefinites/cardinal NPs and bare NPs. When a variable is bound by an indefinite or cardinal NP, the values assigned to it are held fixed, or rigidified, throughout a quantificational context; in (1b), this causes a certain quantity of apples to be held fixed throughout the time intervals introduced by for an hour. By contrast, bare NPs involve flexible existential quantification where assigned values can fluctuate. Given this Rigidity Thesis, (1) through (2c) are easily explained. Rigidity also explains the contrast between (3a) and (3b): the cumulative reading of (3b) is disallowed because rigidity requires the chemistry majors to be the same for all the boys; inserting a distributive operator overrides rigidity but blocks the cumulative reading (Champollion 2016).

Rigidity in PPL: Inspired by Dynamic Plural Logic (DPlL, Brasoveanu 2008, 2013, Henderson 2014) and Team Logic (Väänänen 2007, Dotlačil 2011), we propose Plural Predicate Logic (PPL) to formalize rigidity. In first-order logic, formulas are evaluated relative to single assignments; PPL uses sets of assignments, or quantificational contexts. Building on the pointwise manipulation of assignments defined in (4), we define two types of existential quantifiers: flexible existential quantifiers, as in (5a), and rigid existential quantifiers, as in (5b).

(4) a. \( h[x]g \) := for any variable \( v \), if \( v \neq x \) then \( h(v) = g(v) \)
   b. \( H[x]G \) := \( \forall h \in H \exists g \in G \text{ s.t. } h[x]g \) and \( \forall g \in G \exists h \in H \text{ s.t. } h[x]g \)
   c. \( H[x]!G \) := \( H[x]G \) and \( h(x) = h'(x) \) for any \( h, h' \in H \)

(5) a. \( \exists_{\text{flex}} x [\varphi(\psi)]^G \) iff \( [\varphi \land \psi]^H \) for some \( H \text{ s.t. } H[x]G \)
   b. \( \exists_{\text{rigid}} x [\varphi(\psi)]^G \) iff \( [\varphi \land \psi]^H \) for some \( H \text{ s.t. } H[x]!G \)

Illustrating the approach via for-adverbials: As shown in (6a), for an hour embeds a rigid quantifier which introduces an hour \( t \) via the rigid assignment \( H[t]!G \) keeping it constant across the quantificational context. As in Piñón (2015), we assume that the for-adverbial also introduces a set of proper subintervals \( t' \) of \( t \) that jointly cover \( t \). Since \( t' \) is bound flexibly, each assignment in the context is free to map \( t' \) to a different value. In effect, (6a) unfurls a quantificational context by spreading different subintervals of an hour across its individual assignments.
(6)  a. for an hour(\(\varphi\)) \(\leadsto \exists^{\text{rigid}}t\) [hours(t) = 1](\(\exists^{\text{flex}}t'[t' < t \land t = \ominus t'](\varphi)\))
   
   b. \([x = \ominus y]_{G}\) iff \(\bigoplus\{g(x) : g \in G\} = \bigoplus\{g(y) : g \in G\}\)  
   (\(\bigoplus\) is mereol. sum)

We assume that the for-adverbial passes \(t'\) rather than \(t\) to \(\varphi\), and that ordinary lexical predicates are evaluated at each assignment in the context, requiring \(\varphi\) to hold at each subinterval \(t'\). Since eat is an incremental-theme verb that maps each part of its theme to a different subinterval, the assigned theme values need to covary with the values of \(t'\) across the quantificational context. If the themes are held fixed, the incrementality requirement of eat is violated (Krifka 1998).

This explains the contrasts in (1a) vs. (1b). In (1a), two apples is rigid and requires each assignment to map the variable it binds to the same 2-apple sum, as in (7a). In (1b), apples is a flexible existential, allowing different assignments to map the bound variable to different values, as in (7b). Following Brasoveanu (2013: §2), modified numerals as in (8a) contain a maximalization operator, defined similarly to the rigid existential quantifier. In concert with rigidity, this solves the quantization puzzle: why is e.g. eat less than 3 apples telic? (Zucchi & White 2001)

(7)  a. two\(^x\)[apples](\(\varphi\)) \(\leadsto \exists^{\text{rigid}}x[\exists^{\text{flex}}x[\exists^{\text{apple}}(x) \land |x| = 2](\varphi)](\varphi)\)
   b. \([\exists \emptyset \exists^{\text{apple}}]\)(\(\varphi\)) \(\leadsto \exists^{\text{flex}}x[\exists^{\text{apple}}(x)](\varphi)\)

(8)  a. less than three\(^x\)[apples](\(\varphi\)) \(\leadsto \sigma x[\exists^{\text{flex}}x[\exists^{\text{apple}}(x) \land \varphi \land |x| < 3]](\varphi)\)
   b. \([\sigma x(\varphi \land \psi)]\) iff \([\varphi \land \psi]_{H}^{H'}\) for some \(H\) s.t. \(H[x]_{G}\), and there is no \(H'\) s.t. \(H'[x]_{G}\) and \([\varphi \land \psi]_{H'}^{H'}\) and \(h'(x) < h(x)\) for some \(h' \in H'\) and some \(h \in H\)

In a scenario where John eats apple\(_1\) from 9pm-9:30pm and apple\(_2\) from 9:30pm-10:00pm, (9) fails due to the rigid existential quantifier, which requires \(x\) to refer to the same (sum of) apple(s) throughout the quantificational context (i.e. \(x\) must be eaten repeatedly). Similarly if the object of (9) is replaced by (8a). By contrast, (10) is OK because \(x\) can refer flexibly to different apples.

(9)  John ate two apples for an hour \(\leadsto \exists^{\text{rigid}}t\) [hours(t) = 1](\(\exists^{\text{flex}}t'[t' < t \land t = \ominus t'](\exists^{\text{apple}}(x) \land |x| = 2)](\text{eat}(x, y, t'))))))

(10) John ate apples for an hour \(\leadsto \exists^{\text{rigid}}t\) [hours(t) = 1](\(\exists^{\text{flex}}t'[t' < t \land t = \ominus t'](\exists^{\text{apple}}(x))[(\text{eat}(x, y, t'))])))))

Distributivity: As in DPL, we analyze every \(N\) as launching a separate quantificational context for each \(N\). In PPL, every \(N\) introduces a variable via an assignment maximalization operator \(M\), as in (11a), and distributes over them via an assignment distributive operator \(D\), as in (11b).

(11)  a. \([Mx[\varphi](\psi)]_{G}\) iff \([\varphi \land \psi]^{H}_{H'}\) for \(H\) s.t. \(H[x]_{G}\) and there is no \(H'\) s.t. \(H'[x]_{G}\) where \([\varphi]^{H'}_{H}\) and \(H(x) \subseteq H'(x)\)
   b. \([Dx(\varphi)]_{G}\) iff \([\varphi]_{G}^{G_{x = a}}\) for each \(a\) s.t. \(\exists g \in G. g(x) = a\)

This analysis naturally explains why all NPs can covary in the scope of every \(N\), as in (12). Every day is translated as (13a), and (12) as (13b). Effectively, (13b) splits up a quantificational context into separate contexts, one for each day. Since a flea checks rigidity only within each context, fleas can vary with days; this is more explanatory than Zucchi & White (2001), who stipulate that every day binds either the individual variable of a flea or a reference time variable introduced by a flea.

(12)  John found a flea (on his dog) every day for a year.

(13)  a. every\(^{t'}\)[day](\(\varphi\)) \(\leadsto M_{\varphi}\) [day(t') \land t' ∩ t \(\neq \emptyset\)](\(D_{\varphi}(\varphi)\))
   b. \(12 \leadsto \exists^{\text{rigid}}t\) [years(t) = 1](\(\exists^{\text{flex}}t'[t' < t \land t = \ominus t'](\exists^{\text{flea}}(y) \land t'' \cap t' \neq \emptyset)](\(D_{\varphi}(\exists^{\text{rigid}}y[x = j]\{\exists^{\text{flex}}y[\exists^{\text{flea}}(y)\text{find}(x, y, t'')])))\))
References


