Percentages, relational degrees, and degree constructions
Adam Gobeski and Marcin Morzycki
Michigan State University

A typical (though certainly not universal) assumption is that comparatives and equatives are closely related, differing chiefly in whether their degree relation permits identity (essentially, > vs ≥). This talk examines a curious but revealing asymmetry between comparatives and equatives: percentage factor phrases are possible with both, but interpreted differently in each case. With comparatives, factor phrases are interpreted differentially (i.e., additively), and with equatives, multiplicatively. This reveals that degree addition is an essential, hard-coded part of what it is to be a comparative, but not an equative—which lexicalizes neither addition nor multiplication. In addition, the analysis leads to a notion of “relational” degrees, ones that express measurement of other degrees.

The basic contrast is that (1a) involves a price that is 180% of the price of a Civic, while (1b), it's 80%:

(1) a. 80% more expensive than this Civic
   b. 80% as expensive as this Civic

(2) a. 30% taller than Clyde
   b. 30% as tall as Clyde

Unlike comparatives, equatives can't take standard differential measure phrases such as six feet:

(3) a. $8K more expensive than this Civic
   b. *$8K as expensive as this Civic

The key thing to note is that 80% in (1a) behaves the same way as the standard measure phrase in (3a): it describes the amount by which the price has been exceeded. By contrast, (3b) is the result of a direct modification of the degree of Clyde’s height which is then found equal to Floyd's height. These percentage phrases are internally different as well. A version of (3a) in which a price is made explicit is well-formed, if perhaps hard to parse, but its comparative analogue is clearly ungrammatical:

(4) a. Floyd is 30% of Clyde’s height taller than Clyde.
   b. *Floyd is 30% of Clyde’s height as tall as Clyde.

This suggests that the percentage phrase in (4a) refers to a specific height degree, and that the one in (4b) doesn’t. Together, the facts in (3) and (4) suggest that the percentage phrase is a species of factor phrase, as in (5). That accords with the fact that factor phrases are the only kind of measure phrase equatives seem to support:

(5) Floyd is three times taller than Clyde.

The analytical task, then, is to provide a semantics for the percentage phrase, comparative, and equative that treats percentage phrases like factor phrases, and yields an additive interpretation for comparatives and a multiplicative one for equatives.

Factor (or ratio) phrases aren't well understood. While they appear frequently and in multiple contexts, little work has been done on them and any discussion tends to be brief (as in e.g. Rett 2008). The most notable exceptions are Sassoon (2010a) and Sassoon (2010b). The former discusses factor phrases in terms of measurement theory; she notes that when factor phrases are used, it’s not that specific units are assigned but rather that a ratio measurement between the two objects is established. We will build on that intuition, but of course it alone won’t suffice to yield an account of the contrasts here.
The first analytical step is to provide a semantics for hard-to-parse but conveniently overt comparative cases such as (4). In these cases, the percentage phrase is built out of (what we'll assume is) another degree-denoting expression, Clyde's height. The combination of the two has to denote a degree itself (or in any case whatever measure phrases denote). Thus heart of the measure phrase, in this case 30%, is a function of type (d, d). Assuming a fairly standard denotation for the differential comparative (von Stechow 1984 a.o.) but modified to accord with the 'big DegP' syntax in which DegP is an projection of AP (Abney 1987, Kennedy 1997), the pieces fit together as in (6):

(6) a. \[30\%\] = \lambda d . 30\% \times d
b. \[30\%\] (\[\text{of Clyde's height}\]) = 30\% \times \text{d}_{\text{Clyde}}
c. \[\text{more}\] = \lambda G_{(d, e)} \lambda d \lambda d' \lambda x . \text{max}\{d": G(d'')(x)\} \geq d + d'
d. \[\text{more}\] (\[\text{tall}\])(\[30\%\] of Clyde's height) (\[\text{than Clyde}\])
   = \text{max}\{d": \text{tall}(d'')(x)\} \geq 30\% \times \text{d}_{\text{Clyde}} + \text{d}_{\text{Clyde}}

The equative is helpfully transparent in what it requires—a bare percentage, without the overt complement. The natural way to interpret this is that the 'measure phrase' in such equatives must be of type (d, d) rather than d. This accounts for the incompatibility of equatives with ordinary measure phrases such as six feet. It also predicts their compatibility with factor phrases, because these can straightforwardly be assigned (d, d) denotations.

Before implementing this explicitly, it's worth pausing to recognize the significance of this move. It entails the existence of what might be called relational degrees: objects that behave roughly like measure phrases syntactically, but actually map from one spot on another scale to another. They're not d, of course, but remain 'degrees' in the extended sense that these (d, d) functions can be placed on a scale, just like any other degree, and their distribution resembles ordinary degrees. This is operationalizes the theoretical insight off Sassoon (2010b). It also distinctly echoes the universal scale of (Bale 2008, 2006), and perhaps can be construed as independent evidence for (something like) it.

One simple but inelegant way to implement the approach would be to just adopt a version of the equative morpheme specialized for measure phrases, by analogy to the standard strategy for comparatives:

(7) a. \[\text{as}_{\text{deg}}\] = \lambda G_{(d, e)} \lambda \delta_{(d, d)} \lambda d \lambda x . \text{max}\{d": G(d'')(x)\} \geq \delta(d)
b. \[\text{as}_{\text{deg}}\] (\[\text{tall}\])(\[30\%\])(\[\text{as Clyde}\])
   = \lambda x . \text{max}\{d": \text{tall}(d'')(x)\} \geq 30\% \times \text{d}_{\text{Clyde}}

This would explain the requirement of relational measure phrases. It does render the equative uncomfortably close to the comparative, though and the stipulation seems unenlightening. A more desirable alternative is to suppose that there is no distinct MP-licensing equative denotation, and that the requirement is imposed as a kind of modificational type-shift or rule of composition that applies the relational degree to the standard degree, and can be implemented as in (8):

(8) \text{SHIFT} \overset{\text{def}}{=} \lambda \delta_{(d, d)} \lambda f_{(d, e)} \lambda d \lambda x . f(\delta(d))

This type shift turns out much less simple on the 'small DegP' Bresnan (1973) approach to comparatives, so it may bear on the choice between them.

The larger point is twofold. First, on either implementation, a basic distinction emerges between differential comparatives and equatives. Comparatives lexicalize that they involve addition, whereas in equatives, the operation isn't specified, and comes indirectly from the measure phrase. Second, the grammar of percentage phrases suggests that there are “relational degrees” alongside ordinary ones, and that these play a crucial role in certain flavors of measure phrase.
References
Sassoon, G. 2010a. The degree functions of negative adjectives.